

On the two-photon decay width of the sigma meson

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We shortly report on the two-photon decay width of the light σ -meson interpreted as a quarkonium state. Results are given in dependence on the σ -mass and the constituent mass of the light quark. The triangle quark-loop diagram, responsible for the two-photon transition, is carefully evaluated: a term in the transition amplitude, often omitted in literature, results in destructive interference with the leading term. As a result we show that the two-photon decay width of the σ in the quarkonium picture is less than 1 keV for the physical range of parameters.

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I. INTRODUCTION

The two-photon decay of scalar mesons represents a valuable mechanism to possibly pin down their internal structure (see [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] and references therein). In particular, the transition of the scalar-isoscalar resonance $\sigma \equiv f_0(600)$ into $\gamma\gamma$ has received much attention in the literature. It is commonly believed that a decay width of about 3 - 5 keV would favor a quarkonium interpretation of the $f_0(600)$. In this short work we aim to show that this is not the case: the decay width of a scalar-isoscalar quark-antiquark state, with flavor wave-function $\bar{n}n = \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$ and a mass between 0.4 and 0.8 GeV as favored by recent studies (a mass of about 0.44 GeV is the outcome of [16]), turns out to be smaller than 1 keV for the physical range of parameters. When evaluating the related quark triangle-loop diagram of Fig. 1 care has to be taken concerning gauge invariance, for a comprehensive and detailed analysis we refer to [8]: a (often neglected) term generating a consistent suppression of the decay amplitude is present, as will be discussed in Sections II and III. The omission of this term generates an overestimate of the two-photon decay rate by at least a factor of 2.25. Considering the relevance of this process related to the nature of the σ meson, and in more general of scalar mesons (see for instance [17] and Refs. therein), we consider it as important to stress this point for future considerations about the interpretation of the enigmatic σ -resonance.

The triangle quark loop diagram of Fig. 1 is typical for theories with quarks as effective degrees of freedom [4, 7, 8, 9, 12, 13, 18, 19]. It is evaluated both in the framework of local and nonlocal σ - $\bar{n}n$ vertices. In the local case the Goldberger-Treiman relation on the quark level and the linear realization of chiral symmetry allow to fix the corresponding σ - $\bar{n}n$ coupling constant. In the nonlocal case the finite size of the σ -meson interpreted as a quarkonium state is described by means of a covariant vertex function. The results of local and nonlocal approaches are similar when M_σ is sufficiently below threshold set by the sum of constituent quark masses. Close to threshold care has to be taken in the local case since a second possible problem can arise: the explicit momentum dependence of the σ - $\bar{n}n$ coupling constant cannot be neglected further. This is also a delicate point to be treated with attention.

The present article is organized as follows: in the next two sections we elaborate on the formalism and the results for $\sigma \rightarrow \gamma\gamma$ as based on a local and nonlocal interaction Lagrangian, respectively. In section IV we summarize and give our conclusions.

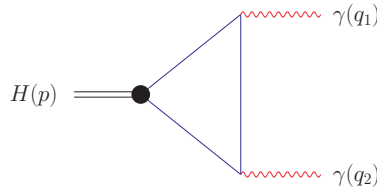


FIG. 1: Quark-loop diagram contributing to $H\gamma\gamma$ decay, where $H = \pi$ or σ .

II. LOCAL CASE

We consider the following local (L) interaction Lagrangian

$$\mathcal{L}_{\text{int}}^{\text{L}}(x) = \frac{g_\sigma}{\sqrt{2}}\sigma(x)\bar{q}(x)q(x) + \frac{g_\pi}{\sqrt{2}}\bar{q}(x)i\gamma_5\vec{\pi}(x)\vec{\tau}q(x) \quad (1)$$

where $q^T = (u, d)$ is the quark doublet of u and d quarks with the constituent mass $m_q = m_u = m_d$ (we restrict to the isospin limit) to be varied between 0.25 and 0.45 GeV, $\sigma(x)$ and $\vec{\pi}(x)$ represent the scalar-isoscalar quarkonium and the isotriplet pion field, respectively, g_σ and g_π are the corresponding coupling constants (which are later related via symmetry and low-energy considerations). We will denote the meson masses by $M_\pi = M_{\pi^0} = 134.9766$ MeV and M_σ , respectively. The latter will be varied between 0.4 and 0.8 GeV.

The decay of $H = \pi^0, \sigma$ into $\gamma\gamma$ is obtained by evaluating the diagram of Fig. 1. The decay width is explicitly given by:

$$\Gamma_{H \rightarrow \gamma\gamma} = \frac{\pi}{4} \alpha^2 g_{H\gamma\gamma}^2 M_H^3, \quad H = \pi^0, \sigma, \quad (2)$$

where $g_{H\gamma\gamma} = g_H N_c Q_H I_H / (2\pi^2)$ is the effective $H\gamma\gamma$ coupling constant, α is the fine structure constant, $N_c = 3$ the number of colors and g_H refers to the coupling constant g_π or g_σ entering in the interaction Lagrangian of Eq. (1). The charge factors $Q_{\pi^0} = \frac{1}{\sqrt{2}}(\frac{4}{9} - \frac{1}{9}) = \frac{3}{9\sqrt{2}}$ and $Q_\sigma = \frac{1}{\sqrt{2}}(\frac{4}{9} + \frac{1}{9}) = \frac{5}{9\sqrt{2}}$ correspond to the flavor wave functions $\pi^0 \equiv \sqrt{\frac{1}{2}}(\bar{u}u - \bar{d}d)$ and $\sigma \equiv \sqrt{\frac{1}{2}}(\bar{u}u + \bar{d}d)$. Finally, the loop integrals I_H [8] corresponding to Fig. 1 are functions of m_q and M_H , which are explicitly given by

$$I_{\pi^0} = I_{\pi^0}(m_q, M_\pi) = m_q \int_0^1 d^3\alpha \delta\left(1 - \sum_{i=1}^3 \alpha_i\right) \frac{1}{m_q^2 - M_\pi^2 \alpha_1 \alpha_2} = \frac{2m_q}{M_\pi^2} \arcsin^2\left(\frac{M_\pi}{2m_q}\right), \quad (3)$$

$$I_\sigma = I_\sigma(m_q, M_\sigma) = m_q \int_0^1 d^3\alpha \delta\left(1 - \sum_{i=1}^3 \alpha_i\right) \frac{1 - 4\alpha_1 \alpha_2}{m_q^2 - M_\sigma^2 \alpha_1 \alpha_2} = \frac{2m_q}{M_\sigma^2} \left[1 + \left(1 - \frac{4m_q^2}{M_\sigma^2}\right) \arcsin^2\left(\frac{M_\sigma}{2m_q}\right)\right]. \quad (4)$$

Note that the only difference between I_{π^0} and I_σ is the term proportional to $-4\alpha_1 \alpha_2$ present in the integral expression of I_σ , which is generally neglected in the literature (that is, the amplitudes of $\pi^0 \rightarrow \gamma\gamma$ and $\sigma \rightarrow \gamma\gamma$ cannot be set equal to each other as done, for instance, in [12, 13]). The presence of the term $-4\alpha_1 \alpha_2$ generates a destructive interference with the first term, which leads to a sizable reduction of the full amplitude I_σ . Quantitatively the ratio of amplitudes is limited by $I_\sigma(m_q, x)/I_{\pi^0}(m_q, x) < 0.667$ for values of $0 < x < 2m_q$ in the region of applicability. Thus, neglecting the additional term in I_σ implies an overestimate of the decay rate $\Gamma_{\sigma \rightarrow \gamma\gamma}$ by at least a factor of $0.667^{-2} = 2.25$, as already indicated in the Introduction. Notice that we compare the decay amplitudes $I_\sigma(m_q, x)$ and $I_{\pi^0}(m_q, x)$ but not the corresponding decay widths: as shown below these will differ consistently because of the dependence on the third power of the meson mass in eq. (2).

Let us now turn to the explicit calculation of decay rates. The Goldberger-Treiman (GT) relation $g_\pi = m_q \sqrt{2}/F_\pi$ with $F_\pi = 92.4$ MeV allows to determine g_π . As an outcome we obtain for the decay width $\Gamma_{\pi^0 \rightarrow \gamma\gamma} = 7.73 - 8.12$ eV for constituent quark masses in the range $m_q = 0.45 - 0.25$ GeV, in good agreement with the experimental result $\Gamma_{\pi^0 \rightarrow \gamma\gamma}^{\text{exp}} = 7.7 \pm 0.5 \pm 0.5$ eV [20]. Only a very weak dependence on m_q is observed.

The linear realization of chiral symmetry implies $g_\sigma = g_\pi$ [8, 12, 19]. Predictions for $\Gamma_{\sigma \rightarrow \gamma\gamma}$ can then be obtained in dependence on the effective quark mass m_q and on M_σ . Note that we limit the parameter space by the relation $M_\sigma < 2m_q$: in fact, only when this condition is met the amplitude I_σ remains real and no unphysical decay of the sigma meson into a quark-antiquark pair is included. Furthermore, the condition $g_\sigma = g_\pi$ can only be employed, if M_σ is safely below the threshold $2m_q$. For $M_\sigma \sim 2m_q$ the momentum dependence of g_σ becomes non-negligible leading to a value for g_σ smaller than the one obtained in the GT limit $m_q \sqrt{2}/F_\pi$; in the next section we illustrate this point in the context of the nonlocal approach.

Table 1: $\Gamma_{\sigma \rightarrow \gamma\gamma}$ in the local case for $m_q = 0.25 - 0.45$ GeV at $M_\sigma = 0.440$ GeV

m_q (GeV)	0.25	0.3	0.35	0.4	0.45
$\Gamma_{\sigma \rightarrow \gamma\gamma}$ (keV)	0.54	0.45	0.41	0.39	0.37

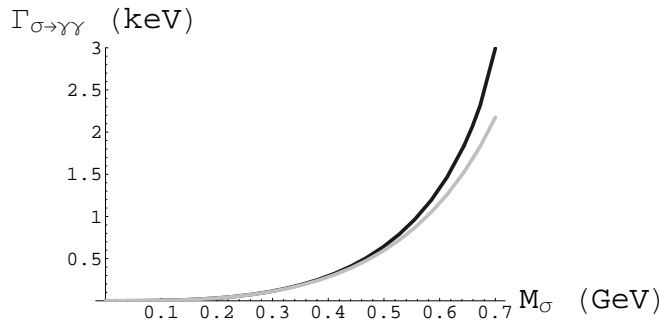


FIG. 2: $\Gamma_{\sigma \rightarrow \gamma\gamma}$ in the local case as function of M_σ for $m_q = 0.35$ (dark) and $m_q = 0.4$ (gray).

In Table 1 we report the results for $\Gamma_{\sigma \rightarrow \gamma\gamma}$ at a fixed pole mass of $M_\sigma = 440$ MeV as favored by recent theoretical and experimental works [16, 20]. The results are weakly dependent on m_q and clearly point to a decay width smaller than 1 keV, when the sigma meson is interpreted as a quarkonium state. Note for example that the omission of the previously discussed term in Eq. (4) implies an overestimated decay width of $\Gamma_{\sigma \rightarrow \gamma\gamma} = 1.18$ keV for a value of $m_q = 0.3$ GeV, to be compared to the correct result of 0.49 keV reported in Table 1.

In Fig. 2 we indicate the dependence of $\Gamma_{\sigma \rightarrow \gamma\gamma}$ on M_σ for values of the constituent quark mass, $m_q = 0.35$ and 0.4 GeV, very often used in phenomenological studies. For values of M_σ safely below threshold (up to 0.5 GeV) results for $\Gamma_{\sigma \rightarrow \gamma\gamma}$ lie below 1 keV and essentially do not depend on the quark mass. For values of M_σ approaching threshold the dependence on m_q becomes more pronounced, where the results for $\Gamma_{\sigma \rightarrow \gamma\gamma}$ eventually grow beyond 1 keV. However, the local approach is no longer applicable for values of M_σ close to threshold, as will be evident from the discussion of the next section.

III. NONLOCAL CASE

Next we study the sigma meson described by the nonlocal (NL) interaction Lagrangian [8]

$$\mathcal{L}_{\text{int}}^{\text{NL}}(x) = \frac{g_\sigma}{\sqrt{2}} \sigma(x) \int d^4y \Phi(y^2) \bar{q}(x+y/2) q(x-y/2), \quad (5)$$

where the delocalization takes account of the extended nature of the quarkonium state by the covariant vertex function $\Phi(y^2)$. The (Euclidean) Fourier transform of this vertex function is taken as $\tilde{\Phi}(k_E^2) = \exp(-k_E^2/\Lambda^2)$, also assuring UV-convergence of the model. The cutoff parameter Λ will be varied between 1 and 2 GeV, corresponding to an extension of the σ of about $l \sim 1/\Lambda \sim 0.5$ fm. Previous studies [21] have shown that the precise choice of $\tilde{\Phi}(k_E^2)$ affects only slightly the result, as long as the function falls off sufficiently fast at the energy scale set by Λ . The coupling g_σ is determined by the so-called compositeness condition $Z_\sigma = 1 - \Sigma'_\sigma(M_\sigma^2) = 0$ [4, 8, 22], where Σ'_σ is the derivative of the σ -meson mass operator given by

$$\Sigma_\sigma(p^2) = -g_\sigma^2 N_c \int \frac{d^4k}{(2\pi)^4 i} \tilde{\Phi}^2(-k^2) \text{tr} [S_q(k+p/2) S_q(k-p/2)], \quad (6)$$

where $S_q(k) = (m_q - \not{k})^{-1}$ is the quark propagator. Note, the compositeness condition is equivalent to the hadron wave function normalization condition in quantum field approaches based on the solution of the Bethe-Salpeter/Faddeev equation [23]. At this level it is clear that g_σ is a function of M_σ . In Fig. 3 we give the dependence of $g_\sigma(M_\sigma^2)$ at $m_q = 0.35$ GeV for cut-off values of $\Lambda = 1$ and 2 GeV, respectively, and indicate the local limit with $g_\sigma = m_q \sqrt{2}/F_\pi$. For low values of M_σ the coupling $g_\sigma(M_\sigma^2)$ is a slowly varying function, values of which for $\Lambda = 1 - 2$ GeV also include the GT limit. However, for values of M_σ approaching threshold $g_\sigma(M_\sigma^2)$ decreases below the local result (see details in Ref. [24]).

We turn to the $\sigma \rightarrow \gamma\gamma$ decay amplitude, where a similar suppression is found. Due to the presence of the vertex function $\Phi(y^2)$ inclusion of the electromagnetic interaction is achieved by gauging the nonlocal interaction Lagrangian (5): in addition to the photon-quark coupling, already present in the local case, in leading order a new vertex arises, where the photon couples directly to the $\sigma\gamma\gamma$ interaction vertex, see [8] for details. In particular, in addition to the triangle diagram of Fig. 1 we have additional diagrams (see Fig. 5 in Ref. [8]) to fully guarantee gauge invariance of the transition amplitude. In practice it is convenient to split the contribution of each diagram into a part which is

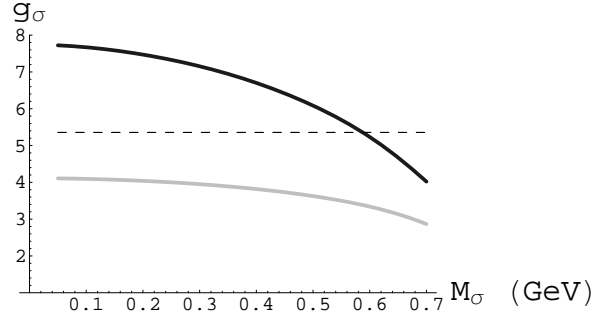


FIG. 3: M_σ -dependence of the coupling $g_\sigma(M_\sigma^2)$ at $m_q = 0.35$ for cut-off values of $\Lambda = 1$ (dark) and 2 GeV (gray). The dashed line corresponds to the GT limit.

gauge invariant and one which is not. The remaining terms, which are not gauge invariant, cancel each other in total and in the further calculation one should only proceed with the gauge invariant terms of the separate diagrams. It was shown [8], that the by far dominant contribution comes from the gauge invariant part of the triangle diagram of Fig. 1. The gauge invariant parts of the other diagrams are strongly suppressed (see discussion in Refs. [8, 9]).

Following [8, 9] the contribution of the gauge-invariant part of the triangle diagram to the two-photon decay width is given by:

$$\Gamma_{\sigma \rightarrow 2\gamma} = \frac{\pi}{4} \alpha^2 M_\sigma^3 \left[\frac{g_\sigma}{2\pi^2} Q_\sigma N_c I_\sigma \right]^2, \quad I_\sigma = I_\sigma^{(1)} + I_\sigma^{(2)}, \quad (7)$$

$$I_\sigma^{(1)} = m_q \int \frac{d^4 k}{\pi^2 i} \tilde{\Phi}(-q^2) \frac{1}{(m_q^2 - p_1^2)(m_q^2 - p_2^2)(m_q^2 - p_3^2)}, \quad (8)$$

$$I_\sigma^{(2)} = -m_q \int \frac{d^4 k}{\pi^2 i} \tilde{\Phi}(-q^2) \frac{\frac{4}{M_\sigma^2} k^2 - \frac{32}{M_\sigma^4} (kq_1)(kq_2)}{(m_q^2 - p_1^2)(m_q^2 - p_2^2)(m_q^2 - p_3^2)}. \quad (9)$$

where q_1 and q_2 are the photon momenta and $p_1 = k + q_1$, $p_2 = k$, $p_3 = k - q_2$, $q = (p_1 + p_3)/2$. The term $I_\sigma^{(2)}$ contributes with opposite sign relative to $I_\sigma^{(1)}$ leading to destructive interference. In the local limit, i.e. $\Lambda \rightarrow \infty$, $I_\sigma^{(2)}$ reduces to the term proportional to $-4\alpha_1\alpha_2$ in (4). Note that in the pion case only a term analogous to $I_\sigma^{(1)}$ contributes.

In Fig. 4 we report the results for $\Gamma_{\sigma \rightarrow \gamma\gamma}$ in the nonlocal case as function of M_σ for $m_q = 0.35$ GeV, taking values of $\Lambda = 1$ and 2 GeV. We also indicate the previous local result. While for small M_σ both approaches, local and nonlocal, agree, for increasing M_σ the nonlocal approach delivers smaller decay rates than the local counterpart, because of the threshold effects described above. The nonlocal results depend very weakly on the value Λ , implying that the numerical values for $\Gamma_{\sigma \rightarrow \gamma\gamma}$ are hardly model dependent. In Table 2 we summarize our results for the two pole masses, $M_\sigma = 0.44$ and 0.6 GeV, choosing different values of m_q both for $\Lambda = 1$ GeV and, in parenthesis, for $\Lambda = 2$ GeV.

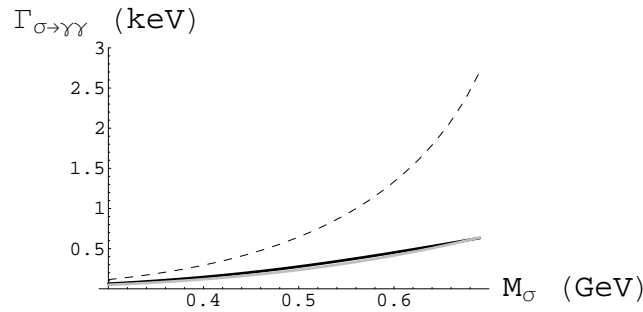


FIG. 4: $\Gamma_{\sigma \rightarrow \gamma\gamma}$ in the nonlocal case as function of M_σ for $\Lambda = 1$ GeV (dark) and 2 GeV (gray). The quark mass is set to $m_q = 0.35$. The upper dashed line corresponds to the local limit evaluated in Section II.

Table 2: $\Gamma_{\sigma \rightarrow \gamma\gamma}$ in the nonlocal case for $m_q = 0.31 - 0.45$ GeV, $\Lambda = 1(2)$ GeV at $M_\sigma = 0.44, 0.6$ GeV.

m_q (GeV)	0.31	0.35	0.40	0.45
$\Gamma_{\sigma \rightarrow \gamma\gamma}$ (keV)	0.238	0.192	0.152	0.124
at $M_\sigma = 0.44$ GeV	(0.196)	(0.159)	(0.127)	(0.105)
$\Gamma_{\sigma \rightarrow \gamma\gamma}$ (keV)	0.529	0.458	0.361	0.294
at $M_\sigma = 0.6$ GeV	(0.512)	(0.415)	(0.327)	(0.267)

The decay widths decrease slowly for increasing quark mass while the dependence on the cutoff is very weak. The numerical analysis shows that

$$\Gamma_{\sigma \rightarrow \gamma\gamma} < 1 \text{ keV for } M_\sigma < 0.7 - 0.8 \text{ GeV.} \quad (10)$$

Again, inclusion of the term $I_\sigma^{(2)}$ of Eq. (9) is crucial to obtain these small decay widths. For instance, omission of this term leads to the incorrect result of $\Gamma_{\sigma \rightarrow \gamma\gamma} = 1.9$ keV for values of $m_q = 0.35$ GeV, $\Lambda = 1$ GeV and $M_\sigma = 0.6$ GeV, which is almost a factor 4 larger than the correct result of 0.458 keV given in Table 2.

In [3], using a Coulomb-like potential, the following expression relating the two-photon decay widths of tensor and scalar states has been derived

$$\Gamma_{\sigma \equiv \bar{n}n \rightarrow 2\gamma}(0^{++}) = k \left(\frac{M_N(0^{++})}{M_N(2^{++})} \right)^m \Gamma_{\bar{n}n \rightarrow 2\gamma}(2^{++}) \quad (11)$$

where $m = 3$. The coefficient k is $15/4$ in a non-relativistic calculation, but becomes smaller ($k \sim 2$) when considering relativistic corrections. Choosing as input $M_N(2^{++}) = 1.275$ GeV and $\Gamma_{\bar{n}n \rightarrow 2\gamma}(2^{++}) = 2.60 \pm 0.24$ keV, Eq. (11) results with $k \sim 2$ in values of $\Gamma_{\bar{n}n \rightarrow 2\gamma}(0^{++}) \sim 0.21$ and 0.54 keV for $M_\sigma = 0.44$ and 0.6 GeV, respectively. These results are close to the corresponding numbers of Table 2. As discussed in [25] different values of the parameter m are obtained for different forms of the quark-antiquark potential: for instance, $m = -1/3$ corresponds to a linear potential. Then, in [25] the value $m = 0$ in Eq. (11) is considered and in Ref. [5] a value $m = 0.3 - 1$, leading to a larger decay width, is discussed. Here notice that our result for a light quarkonium is rather in agreement with the choice $m = 3$ and with $k \sim 2$, see also the model in [9] where an even smaller value of k is obtained.

Notice that we have only considered sigma masses below the constituent quark mass threshold with $M_\sigma < 2m_q$ and masses m_q in the range of 0.25 to 0.45 GeV. In order to go beyond this limit one should (i) either increase the constituent quark mass as done in Ref. [9] where the $\gamma\gamma$ decays of the scalars between 1 and 1.8 GeV have been investigated or (ii) use more general quark propagators which include or mimic confinement. A drawback of these extensions is that the results reached contain a stronger model dependence, thus we do not consider these options here.

IV. CONCLUSIONS

In this work we use the formalism developed in Ref. [8] to study the decay of a scalar quarkonium state into two photons focusing in particular on a technical caveat of this process: a term, not present in the usual $\pi^0 \rightarrow \gamma\gamma$ transition amplitude, is responsible for a sizable suppression of the $\sigma \rightarrow \gamma\gamma$ decay rate. We considered the process $\sigma \rightarrow \gamma\gamma$ in the quarkonium picture both for local and nonlocal approaches. In particular the nonlocal approach allows for a realistic treatment of the finite size effects of the σ -meson. Similar results are obtained in both cases for masses M_σ well below the $2m_q$ threshold. Closer to threshold the two-photon decay width in the local case should be taken with great care, since the momentum dependence of the coupling constant is not properly taken into account. Only the nonlocal result, including finite size effects, is reliable and considerably smaller than for the local case.

Our final result $\Gamma_{\sigma \rightarrow \gamma\gamma} < 1$ keV is smaller the results of dispersive analysis of reaction $\gamma\gamma \rightarrow \pi^0\pi^0$ done in Refs. [5, 26]. Note, that the framework developed in Ref. [26] was based on approach of Ref. [5]. The result of Ref. [5] evaluated at the $M_\sigma = 441$ MeV is $\Gamma_{\sigma \rightarrow \gamma\gamma} = 4.1 \pm 0.3$ keV, while the result of Ref. [26] is around a 40% smaller than that in Ref. [5], mainly due to a smaller $\sigma\pi\pi$ coupling.

When discussing our results it is important to stress that two aspects have not been considered. The first one is the possible role of pion loops. Note, that we consider a scenario where the σ meson is a pure $\bar{q}q$ Fock state and, therefore, the σ couples directly to its constituents – quarks. The coupling with other mesons (e.g. pions) goes via quark loops (a direct coupling of the σ to pions is not present). Inclusion in a such picture of pion loops generating $\sigma \rightarrow \gamma\gamma$ transition can occur as in Fig. 5: the corresponding amplitude is suppressed of a factor $1/N_c$. Our framework is restricted to the one-loop approximation and to the dominant term(s) in the $1/N_c$ expansion. However, being in Nature $N_c = 3$ an explicit calculation of the next-to-leading order would surely be helpful to quantify its contribution

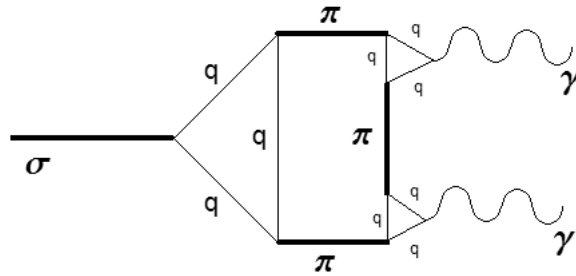


FIG. 5: Pion-loop contribution to the two-photon decay within our model(s). Thick lines refer to mesons, thin lines to quarks.

but goes beyond the scope of present paper and is left as outlook. Notice that, if we propose that the σ meson is not pure $\bar{q}q$ state and there is also two-pion component contribution to the σ meson Fock state, then we should include both possible intermediate states $\bar{q}q$ and 2π contributing to the two-photon transition of the σ . We plan to study the second scenario - σ being mixture of $\bar{q}q$ and 2π - in future.

The second aspect is related to the inclusion of finite-width effects in the evaluation of the full $\gamma\gamma$ -width of the sigma resonance. A careful description of this point would require the precise knowledge of the propagator of the sigma meson dressed by pion clouds: in such a way a definition of the spectral function allowing to integrate over the whole mass range up to 1.27 GeV is possible [24]. Thus, also this aspect is related to pion loops and is not performed here. However, using trial distributions such as Breit-Wigner one and the generalized form of Ref. [24] and varying the mass and the width an increase of few percent is observed. For instance, be the decay rate $\Gamma_{\sigma \rightarrow \gamma\gamma} = 0.458$ keV at $M_\sigma = 0.6$ GeV as in the second column of Table 2: considering a 500 MeV wide Breit-Wigner distribution the integrated width $\Gamma_{\sigma \rightarrow \gamma\gamma}$ reads 0.66 keV. While such effects are surely important in a precision study of the two-photon decay width they does not change the qualitative outcome of the present paper. The corresponding integrated signals decay width(s) reported by [20] are $\Gamma_{\sigma \rightarrow \gamma\gamma} = 3.8 \pm 1.5$ keV and 5.4 ± 2.3 keV, values which are not accepted as average or fit. Note that a confirmation of a large experimental value, contrary to usual belief, does not favor a quarkonium interpretation of the sigma meson. As noted in the PDG2000 [27], the large value for $\Gamma_{\sigma \rightarrow \gamma\gamma}$ could arise from an additional contribution of the broad $f_0(1370)$. A clear experimental determination of the two-photon decay width would certainly help in clarifying the discussion related to the nature of the σ -meson.

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- [1] C. Amsler and N. A. Tornqvist, Phys. Rept. **389**, 61 (2004).
 - [2] N. N. Achasov, S. A. Devyanin and G. N. Shestakov, Phys. Lett. B **108**, 134 (1982) [Erratum-ibid. B **108**, 435 (1982)]; N. N. Achasov, S. A. Devyanin and G. N. Shestakov, Z. Phys. C **16**, 55 (1982); N. N. Achasov and G. N. Shestakov, Phys. Rev. Lett. **99**, 072001 (2007) [arXiv:0704.2368 [hep-ph]].
 - [3] Z. P. Li, F. E. Close and T. Barnes, Phys. Rev. D **43** (1991) 2161.
 - [4] G. V. Efimov and M. A. Ivanov, *The Quark Confinement Model of Hadrons*, (IOP Publishing, Bristol & Philadelphia, 1993).
 - [5] M. R. Pennington, Phys. Rev. Lett. **97**, 011601 (2006); M. R. Pennington, arXiv:hep-ph/0009267; M. R. Pennington, arXiv:hep-ph/0703256.
 - [6] F. E. Close and A. Kirk, Eur. Phys. J. C **21**, 531 (2001) [arXiv:hep-ph/0103173]; W. J. Lee and D. Weingarten, Phys. Rev. D **61**, 014015 (2000) [arXiv:hep-lat/9910008]; M. Strohmeier-Presicek, T. Gutsche, R. Vinh Mau and A. Faessler, Phys. Rev. D **60**, 054010 (1999) [arXiv:hep-ph/9904461]; S. Narison, Phys. Rev. D **73**, 114024 (2006) [arXiv:hep-ph/0512256]; F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Rev. D **72**, 094006 (2005) [arXiv:hep-ph/0509247]; F. Giacosa, T. Gutsche, V. E. Lyubovitskij and A. Faessler, Phys. Lett. B **622**, 277 (2005) [arXiv:hep-ph/0504033]; F. Giacosa, Phys. Rev. D **74** (2006) 014028 [arXiv:hep-ph/0605191].
 - [7] M. K. Volkov, A. E. Radzhabov and V. L. Yudichev, Phys. Atom. Nucl. **66**, 2143 (2003) [Yad. Fiz. **66**, 2193 (2003)] [arXiv:hep-ph/0210306].

- [8] A. Faessler, T. Gutsche, M. A. Ivanov, V. E. Lyubovitskij and P. Wang, Phys. Rev. D **68**, 014011 (2003) [arXiv:hep-ph/0304031].
- [9] F. Giacosa, T. Gutsche and A. Faessler, Phys. Rev. C **71** (2005) 025202 [arXiv:hep-ph/0408085].
- [10] M. A. DeWitt, H. M. Choi and C. R. Ji, Phys. Rev. D **68** (2003) 054026 [arXiv:hep-ph/0306060]; C. Hanhart, Yu. S. Kalashnikova, A. E. Kudryavtsev and A. V. Nefediev, Phys. Rev. D **75** (2007) 074015 [arXiv:hep-ph/0701214]; J. Babcock and J. L. Rosner, Phys. Rev. D **14** (1976) 1286.
- [11] T. Branz, T. Gutsche and V. E. Lyubovitskij, arXiv:0712.0354 [hep-ph].
- [12] E. van Beveren, F. Kleefeld, G. Rupp and M. D. Scadron, Mod. Phys. Lett. A **17** (2002) 1673 [arXiv:hep-ph/0204139]; R. Delbourgo, D. S. Liu and M. D. Scadron, Phys. Lett. B **446** (1999) 332 [arXiv:hep-ph/9811474].
- [13] F. Kleefeld, E. van Beveren, G. Rupp and M. D. Scadron, Phys. Rev. D **66** (2002) 034007 [arXiv:hep-ph/0109158]; M. D. Scadron, F. Kleefeld and G. Rupp, arXiv:hep-ph/0601196.
- [14] M. Schumacher, Prog. Part. Nucl. Phys. **55** (2005) 567 [arXiv:hep-ph/0501167].
- [15] G. Mennessier, P. Minkowski, S. Narison and W. Ochs, arXiv:0707.4511 [hep-ph].
- [16] I. Caprini, G. Colangelo and H. Leutwyler, Phys. Rev. Lett. **96** (2006) 132001 [arXiv:hep-ph/0512364]; G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B **603** (2001) 125 [arXiv:hep-ph/0103088].
- [17] F. E. Close and N. A. Tornqvist, J. Phys. G **28**, R249 (2002) [arXiv:hep-ph/0204205]; R. L. Jaffe, Phys. Rept. **409** (2005) 1 [Nucl. Phys. Proc. Suppl. **142** (2005) 343] [arXiv:hep-ph/0409065]; R. L. Jaffe, arXiv:hep-ph/0701038; R. L. Jaffe, Phys. Rev. D **15** (1977) 267; R. L. Jaffe, Phys. Rev. D **15** (1977) 281; R. L. Jaffe and F. E. Low, Phys. Rev. D **19**, 2105 (1979); M. B. Voloshin and L. B. Okun, JETP Lett. **23**, 333 (1976) [Pisma Zh. Eksp. Teor. Fiz. **23**, 369 (1976)]. N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. B **315** (1989) 465; C. Amsler and F. E. Close, Phys. Lett. B **353**, 385 (1995) [arXiv:hep-ph/9505219]; C. Amsler and F. E. Close, Phys. Rev. D **53**, 295 (1996) [arXiv:hep-ph/9507326]; N. A. Tornqvist, Z. Phys. C **68** (1995) 647 [arXiv:hep-ph/9504372]; M. Boglione and M. R. Pennington, Phys. Rev. D **65** (2002) 114010 [arXiv:hep-ph/0203149]; P. Minkowski and W. Ochs, Nucl. Phys. Proc. Suppl. **121**, 123 (2003) [arXiv:hep-ph/0209225]; H. G. Dosch and S. Narison, Nucl. Phys. Proc. Suppl. **121**, 114 (2003) [arXiv:hep-ph/0208271]; A. H. Fariborz, R. Jora and J. Schechter, Phys. Rev. D **72** (2005) 034001 [arXiv:hep-ph/0506170]; A. H. Fariborz, Int. J. Mod. Phys. A **19** (2004) 2095 [arXiv:hep-ph/0302133]; A. H. Fariborz, Phys. Rev. D **74** (2006) 054030 [arXiv:hep-ph/0607105]; M. Napsuciale and S. Rodriguez, Phys. Rev. D **70** (2004) 094043 [arXiv:hep-ph/0407037]; F. Giacosa, Phys. Rev. D **75** (2007) 054007 [arXiv:hep-ph/0611388]; D. V. Bugg, Eur. Phys. J. C **47** (2006) 57 [arXiv:hep-ph/0603089]; D. V. Bugg, Eur. Phys. J. C **47** (2006) 45 [arXiv:hep-ex/0603023]; N. N. Achasov, arXiv:hep-ph/0609261; M. Ishida, Prog. Theor. Phys. **101** (1999) 661 [arXiv:hep-ph/9902260]; Z. H. Guo, L. Y. Xiao and H. Q. Zheng, arXiv:hep-ph/0610434; J. R. Pelaez, Phys. Rev. Lett. **92** (2004) 102001 [arXiv:hep-ph/0309292]; J. R. Pelaez, Mod. Phys. Lett. A **19** (2004) 2879 [arXiv:hep-ph/0411107]; J. R. Pelaez and G. Rios, Phys. Rev. Lett. **97** (2006) 242002 [arXiv:hep-ph/0610397].
- [18] T. Hatsuda and T. Kunihiro, Phys. Rept. **247**, 221 (1994) [arXiv:hep-ph/9401310].
- [19] S. P. Klevansky, Rev. Mod. Phys. **64** (1992) 649.
- [20] W. M. Yao *et al.* [Particle Data Group], J. Phys. G **33** (2006) 1.
- [21] I. V. Anikin, M. A. Ivanov, N. B. Kulimanova and V. E. Lyubovitskij, Z. Phys. C **65**, 681 (1995).
- [22] S. Weinberg, Phys. Rev. **130**, 776 (1963); A. Salam, Nuovo Cim. **25**, 224 (1962); K. Hayashi, M. Hirayama, T. Muta, N. Seto and T. Shirafuji, Fortsch. Phys. **15**, 625 (1967).
- [23] C. D. Roberts and S. M. Schmidt, Prog. Part. Nucl. Phys. **45**, S1 (2000) [arXiv:nucl-th/0005064]; R. Alkofer and L. von Smekal, Phys. Rept. **353**, 281 (2001) [arXiv:hep-ph/0007355].
- [24] F. Giacosa and G. Pagliara, Phys. Rev. C **76**, 065204 (2007) [arXiv:0707.3594 [hep-ph]].
- [25] M. S. Chanowitz, "Resonances In Photon-Photon Scattering," Invited talk given at 8th Int. Workshop on Photon-Photon Collisions, Jerusalem, Israel, Apr 24-28, 1988. Published in Jerusalem Photon-Photon 1988:0205-229
- [26] J. A. Oller, L. Roca and C. Schat, Phys. Lett. B **659** (2008) 201 [arXiv:0708.1659 [hep-ph]].
- [27] D. E. Groom *et al.* [Particle Data Group Collaboration], Eur. Phys. J. C **15**, 1 (2000).